Substitution of Eq. (7) into Eq. (2) yields the final result

$$E = -(8\pi/3)\boldsymbol{\mu}_N \cdot \mathbf{M}(0), \tag{9}$$

which shows that the nuclear moment $\boldsymbol{\mu}_N$ may be regarded as being subject to the "contact hyperfine field" $(8\pi/3)\mathbf{M}(0)$ arising from the value of the electronic magnetization at the nucleus. According to the considerations which follow Eq. (7), the contact hyperfine field is necessarily equal to the magnetic induction in the interior of a sphere possessing a uniform electronic magnetization.

It remains to comment on the role of those electrons (non-s electrons) whose $\mathbf{M}$ at the nucleus vanishes. The direct interaction of such electrons with the nucleus is not considered to be part of the *contact* hyperfine interaction. Their indirect interaction with the nucleus, on the other hand, is automatically included in Eq. (9) because this interaction results from a modification (e.g., via spin or exchange polarization) of the value of $\mathbf{M}(0)$.

Finally, we note that the replacement of the dynamical variables occurring in Eq. (9) by the corresponding quantum-mechanical operators leads to the spin Hamiltonian

$$\mathcal{H} = (8\pi/3) g_N g_e \beta_N \beta_e \mathbf{I} \cdot \mathbf{s} |\psi(0)|^2, \tag{10}$$

where g_N, g_e, β_N, β_e, $\mathbf{I}$, $\mathbf{s}$, and $|\psi(0)|^2$ denote, respectively, the nuclear and electronic g factors, the nuclear magneton, the absolute value of the Bohr magneton, the nuclear and electronic spin operators, and the position probability density of an s electron at the nucleus. The observable values of the contact hyperfine interaction energy are the eigenvalues of $\mathcal{H}$. For $s=1/2$ and $I=1$, the example discussed by Fermi,[2] the diagonalization of $\mathcal{H}$ yields a quartet and a doublet.

"Meta" Relativity

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In pre-relativity times, Thomson, Heaviside, and Sommerfeld, among others, had examined questions arising from the assumption that a particle may move faster than the velocity of light *in vacuo*. Such a hypothesis is reexamined in the framework of classical (nonquantum) theory of special relativity.

"There was a lady named Bright
Who traveled faster than Light..."

INTRODUCTION

ONE of the favorite topics for luncheon conversations among physicists is the speculation whether the existence of a class of particles, *created* with a velocity $v>c$, may be hypothesized. One would then be dealing with three distinct classes of particles. The first two are conventional. Class I includes all particles which travel at velocities smaller than the velocity of light. Class II is made up of particles which can only exist when traveling with the velocity of light. The third class would then comprise the hypothetical particles which are created at superluminary velocities. In this paper, the implications of such a hypothesis are investigated with a rigour somewhat greater than gastronomic, to see if there could possibly be any physical content in such a generalization. An attempt is also made to devise an experiment by which the existence of such a third class could be tested directly. It should be pointed out that in the present discussion only the classical (nonquantum) aspects of the problem are examined, since they stand for themselves. In this sense, this paper purports to continue the discussion on the "Überlichtgeschwindigkeitsteilchen" (particles of super-luminary velocities) elaborated by Sommerfeld[1,2] in

[1] A. Sommerfeld, K. Akad. Wet. Amsterdam. Proc. **8**, 346 (1904) (translated from Verslag v. d. gewone vergadering d. Wis-en Natuurkundige Afd., November 26, 1904, Dl. XIII). Earlier, less exhaustive, treatments include: J. J. Thomson, Phil. Mag. **28**, 13 (1889); O. Heaviside, *Electrical Papers* (MacMillan and Company, London, 1892), Vol. II, Chap. 47, p. 497; Th. Des Coudres, Arch. Néerland. Sci. [II] **5**, 652 (1900).

[2] A. Sommerfeld, Nachr. Ges. Wiss. Göttingen, pp. 201–235, February 25, 1905.

pre-relativistic times, except that in the present examination the postulates of special relativity are strictly adhered to. A field-theoretical treatment of the hypothesis will be published at a later date.

MASS SHELL CONSIDERATIONS

Let us start by writing down the two criteria which a consistent relativistic theory should satisfy.

(a) In any frame of reference the energy of a particle must be positive.

(b) Laws of particle dynamics must be independent of frame of reference.

It is conventional to satisfy both these demands by requiring the particles to be characterized by energy-momentum four-vectors lying inside, or on, the forward light cone. Events viewed from a different frame are then described by new four-vectors which are transforms of the original ones.

The energy-momentum four-vectors associated with the first two classes satisfy the invariant relation

$$E^2 - p^2c^2 = m_0{}^2c^4. \tag{1}$$

For $m_0{}^2c^4 > 0$, or class I particles, Eq. (1) represents a two-sheeted hyperboloid of revolution around the E axis. A three-dimensional model of such an $(E,\mathbf{p})$ surface is shown in Fig. 1(a). The criterion (a) above restricts the (E,p) coordinates of a particle to lie on the positive energy sheet, but all points on this sheet can be transformed into one another under proper Lorentz transformations. It should be noted that while there exists a Lorentz frame in which the class I particle has zero momentum, as a result of the nonzero mass there exists no frame in which such a particle has zero energy.

For class II particles, with $m_0{}^2 = 0$, the $(E,\mathbf{p})$ surface becomes a cone of revolution about the E axis, as shown in Fig. 1(b). It may appear at first that only the upper cone has physical significance, and that a Lorentz transformation can take a point on the upper cone only into another point on the upper cone. Any transformation (e.g., a reflection) into a point on the lower cone appears to introduce a particle traveling with negative energy, a situation excluded by criterion (a). A further examination (see examples below) reveals, however, that such a transformation implies the photon traveling backward in time. Taken by itself this conclusion appears nonsensical, but when taken together with the negative energy result, it leads to a simple physical reinterpretation. The photon, which in the first system carried positive energy from $(x,t) = (x_1,t_1)$ to $(x,t) = (x_2,t_2)$ with $t_2 > t_1$, would appear to the other observer not as a weird negative energy particle traveling backward in time, but as a positive energy particle traveling forward in time, but going in the opposite direction. Thus, the reinterpretation brings the events back into the fold of ordinary phenomena.

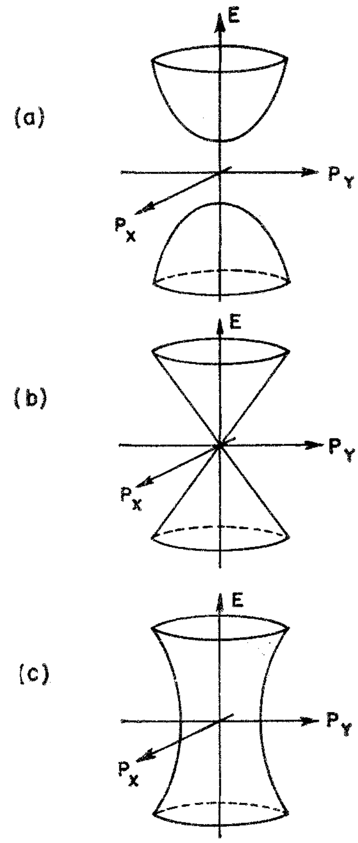

FIG. 1. Three-dimensional models of the $(E,\mathbf{p})$ surfaces described by the invariant relation $E^2 - p^2c^2 = m_0{}^2c^4$, (a) for the class I particles with $m_0{}^2c^4 > 0$, (b) for the class II particles with $m_0{}^2c^4 = 0$, and (c) for the class III particles with $m_0{}^2c^4 < 0$.

The above reinterpretation acquires particular significance when a third class of particles, for which $v > c$, is postulated. For such a particle to have physical significance its energy

$$E = m_0c^2/[1-(v/c)^2]^{\frac{1}{2}}, \tag{2}$$

and its momentum

$$p=m_0v/[1-(v/c)^2]^{\frac{1}{2}}, \quad (3)$$

must be real. This implies imaginary "rest mass" for this particle, which may seem to disqualify the whole idea right from the start. One should recall, however, that in classical mechanics the mass m_0 is a parameter which *cannot* be measured directly even for slow particles. As Max Jammer[3] puts it, mass "does not do what it does because it is what it is, but it is what it is because it does what it does." Only energy and momentum, by virtue of their conservation in interactions, are measurable, therefore must be real. Thus the imaginary result for the rest mass of the hypothetical "meta" particles offends only the traditional way of thinking, and not observable physics.

On similar grounds, one can resolve the question of proper length L_0 and proper time T_0. Only those quantities which the observers can measure must be real. This means that

$$L=L_0/[1-(v/c)^2]^{\frac{1}{2}} \quad (4)$$

and

$$T=T_0[1-(v/c)^2]^{\frac{1}{2}} \quad (5)$$

must be real. In turn, this implies that for particles with $v>c$ the proper length L_0 and proper time T_0 are imaginary. Any objection to this conclusion is overruled on the grounds that L_0 and T_0 are not accessible to measurement by an observer, who by definition must belong to class I.

Imaginary mass, or $m_0^2<0$, of the class III particles implies that the $(E,\mathbf{p})$ surface described by Eq. (1) is now a single-sheeted hyperboloid of revolution around the E axis, as shown in Fig. 1(c). If the framework of the special theory of relativity is preserved, then all points on the sheet can be transformed into each other under proper Lorentz transformations. The feature of the single-sheeted hyperboloid of not being bounded in either the $+E$ and $-E$ direction appears to introduce the possibility of having infinite energy sources, which would violate a fundamental concept of physics, according to which no such sources can exist. This question is discussed in the third example below. To facilitate this discussion, two simpler cases are examined first.

EXAMPLES

Below, three examples are discussed in which the reinterpretation of phenomena involving class III negative-energy particles is explored in detail. It is shown that the time reversal which always accompanies propagation of negative-energy particles in effect reintroduces a bound for the energy. Before we discuss the specific cases below, let us recall that, according to the original criteria, various observers must agree on the identity of the physical laws but *not* on the description of specific events. Only the physical laws, and not the description of any given phenomenon, must remain invariant as we pass from one frame of reference to another.

For particles of class III this description can be so chosen that in any one frame only particles of positive energy appear. Such reinterpretation is made possible by the fact that particles in the negative-energy portion of the $(E,\mathbf{p})$ hyperboloid appear to travel backward in time. These two facts in effect restore positive definiteness of energy for all observers even though the hyperboloid is single-sheeted so that all points on it can be transformed into each other under proper Lorentz transformations.

Let us elaborate on this point by examining a special case.

(1) Assume that the following events take place in a reference frame x. The source S_1 at $x_1=0$ emits a particle with $v>c$ at time $t_1=0$ and the sink S_2 at x_2 absorbs it at a time $t_2(t_2>t_1)$ (see Fig. 2). Consider another frame x' in which the time component of the interval becomes negative as shown in Fig. 3. In this x' frame the energy is also negative. Therefore as viewed from the frame x', the particle moves with negative energy

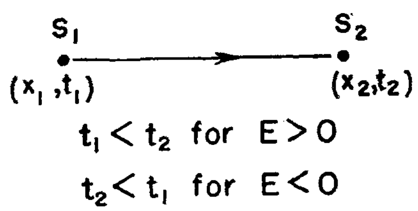

FIG. 2. Interchange of the roles of source and sink for class III particles.

[3] Max Jammer, *Concepts of Mass in Classical and Modern Physics* (Harvard University Press, Cambridge, Massachusetts, 1962), p. 153.

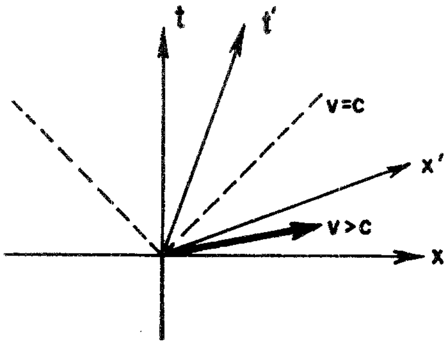

FIG. 3. A displacement of a class III particle from (0,0) to (x,t) acquires a negative time component in a primed system. The situation can be interpreted as an ordinary displacement from (x',t') to (0,0).

from S_1 to S_2 but in such a way that $t_2'<t_1'$. Removal of negative energy from S_1 and addition of negative energy to S_2 implies an increase of energy of S_1 and decrease of energy of S_2 by the same amount. Significantly, the decrease of energy of S_2 occurs *before* the increase of energy of S_1. Therefore the observation made in system x' will show that S_2 is the source and S_1 the sink. According to an observer in system x', a positive energy bundle moves, in effect, from S_2 to S_1. The consistency of the interpretation of physical phenomena is thus restored.

(2) As a second example, consider the collision of a particle of class III ($m_0^2<0$) with a particle of class I ($M_0^2>0$). Let their initial momenta and energies be, respectively, $(\mathbf{p}_1,E_1)$ and $(\mathbf{p}_2,E_2)$, and the final ones $(\mathbf{p}_1',E_1')$ and $(\mathbf{p}_2',E_2')$. Since the hypothesis of the existence of class III particles is made with the full understanding that the accepted physical principles are to remain unaffected, the conservation of momentum and of energy and the invariance of the "length" of the energy-momentum four-vectors, impose the conditions

$$\begin{aligned} \mathbf{p}_1+\mathbf{p}_2&=\mathbf{p}_1'+\mathbf{p}_2' \\ E_1+E_2&=E_1'+E_2' \\ E_1^2-p_1^2c^2&=E_1'^2-p_1'^2c^2=m_0^2c^4<0 \\ E_2^2-p_2^2c^2&=E_2'^2-p_2'^2c^2=M_0^2c^4>0. \end{aligned} \tag{6}$$

If we assume that in our frame of reference the four vectors have positive time components, i.e., all particles have positive energy, the collision will appear simply as an elastic two-particle collision. A question arises when the process is viewed from another system, which moves with respect to ours, in such fashion as to have the class III particle appear to carry negative energy. From the example above, however, we know that such a particle will also appear to be moving backward in time. Hence the observer in the other system will find it natural to describe the process as a fusion of three particles with four-momenta $(\mathbf{p}_1,E_1)$, $(\mathbf{p}_2,E_2)$ and $(-\mathbf{p}_1',-E_1')$ to form a particle with four-momentum $(\mathbf{p}_2',E_2)$. Such reinterpretation leaves the energy-momentum conservation and the mass-shell restrictions unchanged, although the other observer will find it more natural to write the relationships (6) as

$$\begin{aligned} \mathbf{p}_1+\mathbf{p}_2+(-\mathbf{p}_1')&=\mathbf{p}_2' \\ E_1+E_2+(-E_1')&=E_2 \\ E_1^2-p_1^2c^2&=(-E_1)^2-(-p_1')^2c^2=m_0c^2<0 \\ E_2^2-p_2^2c^2&=E_2'^2-p_2'^2c^2=m_0c^2>0. \end{aligned} \tag{7}$$

A graphical representation of the two points of view is given in Fig. 4.

(3) As a final example, let us consider the simultaneous collision of two particles of class III with a particle of class I. Let their initial momenta and energies be, respectively,

$$\begin{aligned} (\mathbf{p}_1,E_1)&=[c(-m_0^{2\prime})^{\frac{1}{2}},0,0,0] \\ (\mathbf{p}_2,E_2)&=[-c(-m_0^{2\prime})^{\frac{1}{2}},0,0,0] \\ (\mathbf{p}_3,E_3)&=(0,0,0,E). \end{aligned} \tag{8}$$

A possible result of collision is

$$\begin{aligned} (\mathbf{p}_1',E_1')&=(\mathbf{p}',-\epsilon) \\ (\mathbf{p}_2',E_2')&=(-\mathbf{p}',-\epsilon) \\ (\mathbf{p}_3',E_3')&=(0,0,0,E+2\epsilon), \end{aligned} \tag{9}$$

where $\epsilon=(p'^2c^2+m_0^2c^4)^{\frac{1}{2}}>0$ can be arbitrarily large by having p' large. It would then appear that the class I particle could be imparted an arbitrarily large amount of energy. If the "meta"

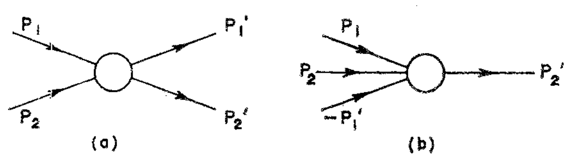

FIG. 4. Collision of a class III particle with a class I particle as viewed (a) from a frame in which only positive energy particles are seen, (b) from a frame in which a negative energy particle seems to appear. The latter situation is reinterpreted as fusion of three particles.

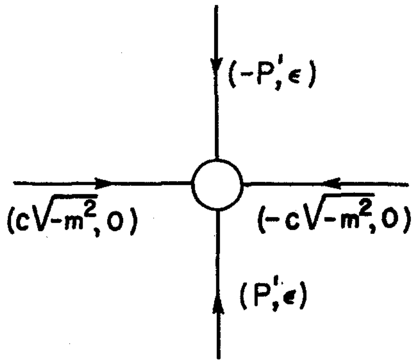

FIG. 5. Simultaneous collision of two class III particles with a class I particle is interpreted as fusion of *four* class III particles with the class I particle in a frame in which the two class III particles seem to acquire negative energies in the collision.

particles could indeed serve as an infinite source of energy, the notion of their existence would violate the fundamental physical concept which excludes the existence of such sources. The resolution of this apparent contradiction is again achieved by properly reinterpreting the process. In any frame in which the energy of a class III particle appears negative ($-\epsilon$ above) the particle will also appear to be moving backwards in time. Thus the time sequence of the process will be such that the observer will see a fusion of *four* particles with the class I particle, see Fig. 5. The increase of energy of the class I particle by 2ϵ is accounted for by the two fusing particles which bring in an energy ϵ each from external sources.

VELOCITY ADDITION

Further light can be shed on the properties of the hypothetical "meta" particles by considering the question of velocity addition. Let u and v be the velocities of a particle as measured by two observers O_1 and O_2, respectively, whose relative velocity is w. Our assumption that class III particles obey the invariance of Eq. (1) implies that they comply with the relativistic law of velocity addition

$$v=(u+w)/[1+(uw/c^2)]. \tag{10}$$

The consequences of this generalization are graphically represented in Fig. 6, where v is plotted as a function of w, the relative velocity of the observers, for the three special cases of $u<c$, $u=c$ and $u>c$. Since all observers belong to class I, the range of w is restricted to $|w|<c$.

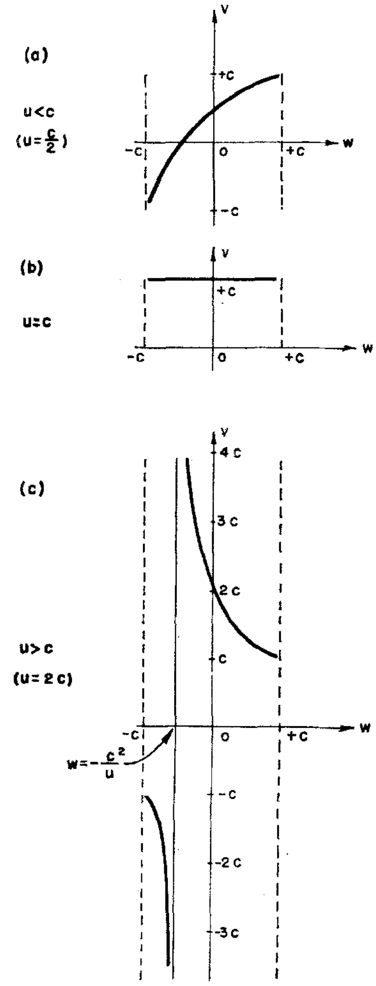

FIG. 6. Graphical representation of the assumption that the relativistic velocity addition $v=(u+w)/(1+uw/c^2)$ holds (a) for class I particles, (b) class II particles, (c) class III particles. If u is the velocity of a particle in our frame of reference, the graphs indicate the velocity v of the same particle as measured by an observer who moves with respect to our frame with a velocity w. Graphs are restricted to $|w|<c$ since all observers are assumed to belong to class I.

Parts (a) and (b) of Fig. 6 represent the familiar situations as encountered with class I and class II particles. Part (c) of Fig. 6 brings into focus some of the striking properties of class III particles. First of all, it should be noted that the role which c plays as the limiting velocity for particles of class I is still with us in class III, except that c is the *lower* limit for the velocity here. This result reflects the fact that the energy-momentum hyperboloid of Fig. 1(c) does not comprise any points with $p^2<m_0^2c^2$. There is no

Lorentz frame in which the meta particle would travel with a velocity equal to or smaller than c.

As the velocity of the observer O_2 relative to O_1 approaches $w=-c^2/u$, the velocity of the meta particle, according to O_2, tends to infinity. Such a result in itself would suffice to disqualify the hypothesis, because it appears to violate the postulate that no energy propagation can take place with infinite velocity. Yet when the energy of a particle is evaluated in the system in which the particle velocity v tends to infinity it is seen that the energy $E=m_0c^2/[1-(v/c)^2]^{\frac{1}{2}}$ tends to zero, so that the above principle, too, stays inviolate. It is interesting to note that the situation of $w=-c^2/u$ for class III particles corresponds to the state of rest of class I particles. The latter have zero momentum and minimum energy, the former have zero energy and minimum momentum, $p^2=m_0v/[1-(v/c)^2]^{\frac{1}{2}}=E^2/c^2-m_0^2c^2=-m_0^2c^2(>0)$. In terms of the energy-momentum space, the $w=-c^2/u$ situation corresponds to a Lorentz transformation into a point lying on the $E=0$ girth of the hyperboloid in Fig. 1(c).

DETECTION

The only sure way to ascertain the physical content of the hypothesis is to detect a meta particle. Assuming the hypothetical class III particles to carry electric charge, a possible avenue for their discovery may lie in the Čerenkov effect. Simple geometric arguments indicate that the coherence condition[4] which determines the unique angle of emission of Čerenkov radiation remains in force for class III particles. This suggests that class III particles could be clearly distinguished from the class I particles by the Čerenkov angle which for the former must always be greater than the limiting angle of the latter. The question of the radiation output, however, is not so straightforward. The frequency cut-off, which in the case of class I particles leads to a finite value of energy loss per unit length, cannot be used here and only a detailed re-examination of the formalism can lead to a prediction of the intensity of the Čerenkov radiation resulting from energy loss of meta particles.

Qualitative considerations seem to indicate that a meta particle losing energy in a medium would actually undergo an acceleration.[2] This can also be seen from Figs. 1(a) and 6(c), which show that for class III particles loss of energy implies increase of velocity. Whereas an ordinary class I particle upon loss of momentum stops with zero velocity and finite rest energy, a meta-particle upon loss of energy disappears with infinite velocity but finite momentum. As the energy of the meta-particle decreases, the Čerenkov angle would go to 90°. Has any one of you gentlemen discarded a set of data on such account? It may have been caused not by faulty electronics, as you assumed, but by a shower of meta particles!

CONCLUSION

At least in one respect, the speculations above have proved very successful. When introduced by the way of problems or illustrations in an introductory special relativity course, they have invariably led to lively and penetrating debates among the students.

ACKNOWLEDGMENTS

The authors wish to acknowledge numerous stimulating discussions with many of their academic friends whom they have tried to convince, not always with success, of the consistency of the above hypothesis.

[4] J. V. Jelley, *Čerenkov Radiation* (Pergamon Press, London, 1958), p. 5, and references therein.